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On isolated conformal fixed points and noncritical string theory

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Abstract: We search for the gravity description of unidentified field theories at their conformal fixed points by studying the low energy effective action of six dimensional non-critical string theory. We find constant dilaton solutions by solving both the equations of motion and BPS equations. Our solutions include a free parameter provided by a stack of uncharged space filling branes. We find several $AdS_p \times S^q$ solutions with constant radii for AdS_p and S^q. The curvature of the solutions are of the order of the string scale.

Keywords: String Duality, Conformal Field Models in String Theory, AdS-CFT and dS-CFT Correspondence.

Dedicated to Farhad Ardalan on the occasion of his 65th birthday.

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Contents

1. Introduction

The conjectured duality of string and gauge theories found its first explicit realization in the context of AdS/CFT correspondence [1]–Witten:1998qj. This correspondence is based upon the analysis of D3 branes in IIB string theory and its low energy effective SUGRA in the limit where the branes are replaced by their RR fluxes. The original form of AdS/CFT and its modifications relate critical IIB string theories and four dimensional conformal gauge theories in the complementary regimes of the string sigma model and gauge couplings.

This strong/weak nature of the duality enables one to obtain lots of information about some strongly coupled gauge theories by studying the corresponding low energy supergravities. However, this ideology breaks down when the coupling of the CFT cannot be varied arbitrarily. Examples of such CFT's can be found in certain $\mathcal{N}=1$ gauge theories [4].

This and other considerations have motivated the study of noncritical strings in search for the gravity duals of gauge theories that have isolated fixed points in their renormalization group flow [5].[1] Such theories are suggested to be dual to noncritical strings with target spaces of the form $AdS_p \times S^q$.[2]

By studying the one loop β function of noncritical sigma models, it has been shown that the corresponding low energy effective actions admit solutions of this form with constant radii for the AdS_p and S^q spaces. In fact these solutions have curvatures of the order of the

[1]For further studies in this direction see [6]–[17].

[2]This scenario has mainly studied for the noncritical string in dimensions less that 10. Noncritical string for $d > 10$ has also been studied, for example, in [18] where it was shown that in comparison with the $d < 10$ case it exhibits new feature, such as a dilaton potential with nontrivial minima at small cosmological constant and d-dimensional string coupling.

string scale. Therefore the one loop approximation becomes inadequate and higher loops must be taken into account. One expects that in general this will destroy the $AdS \times S$ structure of the solutions but it is believed that this is not the case and the corrections will simply modify the corresponding radii of the solution.

This idea was reinforced in [19] where several simple noncritical $AdS_p \times S^q$ sigma models were suggested by demanding a conformally invariant worldsheet theory. Although the all order invariance was not proved, several arguments were presented in favor of that. It was also shown that the sigma models should necessarily have κ symmetry to allow for renormalizability and thus are, as in the critical case, completely integrable.

It was also mentioned [19] that in order to get a gravity solution in the form of $AdS_p \times S^q$ with a free parameter, one may add flavors to the corresponding gauge theory.

Following this idea, the authors of [20] considered 3 brane solutions in presence of space filling $D5/\bar{D}5$ brane pairs in six dimensional noncritical string theory and were able to find an $AdS_5 \times S^1$ solution with fixed radii for AdS_5 and S^1. In this construction the space filling $D5/\bar{D}5$ branes play the role of the flavors in the corresponding gauge theory. Therefore getting an AdS_5 solution could be expected because the solution must correspond to $\mathcal{N}= 1$ superconformal gauge theory which necessarily includes matter.

By arguments regarding stability of the solution and also the open string tachyons on the $D5$-branes, the free parameter was restricted to a certain range which was suggested to be identical to the Seiberg conformal window of the corresponding gauge theory. It was also speculated that Seiberg's electric-magnetic duality is related to T-duality on the S^1.

The supergravity solutions for noncritical string in $d > 1$ dimensions have also been obtained in [22] where several solutions in the form of $AdS_p \times S^q$ were presented. In this paper the authors have considered cases where only RR or NS forms are nonzero. As a result they have not been able to find the solution of [20] where the flavors play an essential role.

It is the aim of this paper to further study more general brane solutions in six dimensional string theory. With the goal of a better understanding of unidentified field theories with conformal fixed points, we mainly focus on solutions with a constant dilaton.

We keep the uncharged space filling branes in the setup to have a free parameter in our solutions. These can be $D5/\bar{D}5$ branes in the case of type B and non BPS 5 branes in the case of type A theories.[3] In particular it also enables us to find solutions like $AdS_3 \times S^3$ which are not possible in the absence of flavors. Apart from solving the standard equations of motion, we insist on finding solutions from the BPS equations, whenever possible, and thus meet the necessary requirement for supersymmetry of solutions.

The organization of the paper is as follows. In section 2 we introduce the general setup of the problem and give a brief review of some of the known results. In section 3 we study gravity solutions in the presence of N_f space filling uncharged brane. In the subsequent two sections 4 and 5 we generalize to the cases where either an RR or an NS charge is turned on. In section 6 we give a second derivation of the solutions by using BPS equations. We conclude by discussions.

[3]We thank Ingo Kirsch for a comment on this point

2. Basic Setup

In this section we shall fix our notations and the basic setup to be used in the next sections. Our starting point is six dimensional noncritical string theory in the presence of different fluxes. In general the theory could contain N_f space filling uncharged branes, NS three form H_3 and nonzero RR k-forms F_k. For this general background the bosonic part of the effective action is given by

$$\begin{aligned} S = &\int dx^6 \sqrt{-g}\Big[e^{-2\phi}\Big(R + 4(\partial\phi)^2 + c\Big) - 2N_f e^{-\phi}\Big] - \\ &-\frac{1}{2}\int dx^6\sqrt{-g}\Big[\frac{e^{-2\phi}}{3!}H_3^2 + \sum_k \frac{1}{k!}F_k^2\Big], \end{aligned} \tag{2.1}$$

where the action is written in the string frame. We work in units where $\alpha' = 1$ such that the cosmological constant term which is the characteristic of the non-critical theory will be

$$c = \frac{10-d}{\alpha'} = 4\,. \tag{2.2}$$

We will keep this constant in our calculations to trace the effect of non-criticality.

Let us consider the following ansatz for the metric in the string frame

$$ds^2 = d\tau^2 + e^{2\lambda(\tau)}dx_n^2 + e^{2\nu(\tau)}d\Omega_k^2\,. \tag{2.3}$$

where τ is the radial coordinate, dx_n^2 is an n dimensional flat space which is associated with the world volume of a D_{n-1} brane and $d\Omega_k^2$ is a k sphere and $n + k + 1 = 6$. For the RR form we take the magnetic ansatz $F_k \sim \omega_k$ with ω_k being the volume form of S_k. This will describe a $(4-k)$ D-brane. Similarly for the NS form we take $H_3 \sim \omega_3$ which will give an NS1 brane. It is clear that the NS three form can contribute only when $k = 3$.

In the following we will only study one nonzero charge, NS or RR, at a time. We will thus be considering D_p-branes with $p = 1, 2, 3, 4$ and NS1 brane. The relations obtained below will be in fact the ones found in [22] which are modified due to the uncharged space filling branes.

Plugging (2.3) into the action (2.1) and doing the possible simplifications one arrives at the following form for the action

$$\begin{aligned} S = &\int d\rho\Big[-n(\lambda')^2 - k(\nu')^2 + (\varphi')^2 + ce^{-2\varphi} + k(k-1)e^{-2\nu-2\varphi}\Big] - \\ &-\int d\rho\Big[N^2 e^{n\lambda-k\nu-\varphi} + Q^2 e^{-2k\nu-2\varphi} + 2N_f e^{\frac{1}{2}(n\lambda+k\nu)-\frac{3}{2}\varphi}\Big], \end{aligned} \tag{2.4}$$

where prime denotes the derivatives with respect to ρ with $d\tau = -e^{\varphi}d\rho$. N and Q are proportional to the number of D_{4-k} and NS1 branes respectively and φ is defined as

$$\varphi = 2\phi - n\lambda - k\nu\,. \tag{2.5}$$

The equations of motion derived from this action read

$$
\begin{aligned}
\lambda''_\rho - \frac{1}{2}N^2 e^{2n\lambda-2\phi} - \frac{1}{2}N_f e^{2n\lambda+2k\nu-3\phi} &= 0\,, \\
\nu''_\rho - (k-1)e^{2n\lambda+2(k-1)\nu-4\phi} + \frac{1}{2}N^2 e^{2n\lambda-2\phi} + Q^2 e^{2n\lambda-4\phi} - & \\
-\frac{1}{2}N_f e^{2n\lambda+2k\nu-3\phi} &= 0\,, \\
\phi''_\rho + \frac{c}{2}e^{2n\lambda+2k\nu-4\phi} - \frac{1}{4}(n+1-k)N^2 e^{2n\lambda-2\phi} + \frac{1}{2}(k-1)Q^2 e^{2n\lambda-4\phi} - & \\
-\frac{1}{4}(n+k+3)N_f e^{2n\lambda+2k\nu-3\phi} &= 0\,.
\end{aligned} \tag{2.6}
$$

To simplify the equations we have written them in terms of ϕ instead of φ. There is a fourth equation, known as the zero energy constraint, which is in fact the equation of motion for $g_{\tau\tau}$[4]

$$
\begin{aligned}
& n(\lambda'_\tau)^2 + k(\nu'_\tau)^2 - (\varphi'_\tau)^2 + c + k(k-1)e^{-2\nu} - \\
& -N^2 e^{n\lambda - k\nu+\varphi} - Q^2 e^{-2k\nu} - 2N_f e^{\frac{1}{2}(n\lambda+k\nu+\varphi)} = 0\,.
\end{aligned} \tag{2.7}
$$

There is a method for finding first order differential equations of motion instead of the second order ones obtained above. These equations are known as BPS equations of motion and the method is based on defining a superpotential W from which the equations are derived. Satisfying the BPS equations is a necessary but not sufficient condition for the solution to be supersymmetric. The method works as follows:

Consider an action of the form

$$
S = \int d\rho \Big(-\frac{1}{2} G_{ab}(f) f^{a\prime} f^{b\prime} - V(f) \Big)\,. \tag{2.8}
$$

If one can find a superpotential $W(f)$ such that

$$
V(f) = \frac{1}{8} G^{ab} \partial_a W \partial_b W \tag{2.9}
$$

then the BPS equations of motion are written as

$$
f^{a\prime} = \frac{1}{2} G^{ab} \partial_b W\,. \tag{2.10}
$$

It is easy to see that the effective action (2.4) can be recast into the above form. Doing so, we find that

$$
G_{\lambda\lambda} = 2n\,, \qquad G_{\nu\nu} = 2k\,, \qquad G_{\varphi\varphi} = -2\,, \tag{2.11}
$$

and the potential reads

$$
V = -(c + k(k-1)e^{-2\nu})e^{-2\varphi} + N^2 e^{n\lambda - k\nu - \varphi} + Q^2 e^{-2k\nu - 2\varphi} + 2N_f e^{\frac{1}{2}(n\lambda + k\nu - 3\varphi)}\,. \tag{2.12}
$$

The relation (2.9) will take the following form

$$
\frac{1}{n}(\partial_\lambda W)^2 + \frac{1}{k}(\partial_\nu W)^2 - (\partial_\varphi W)^2 = 16V\,. \tag{2.13}
$$

[4]Note that the derivatives in (2.6) and (2.7) are with respect to ρ and τ respectively.

and finally the BPS equations are written as

$$\lambda'_\rho = \frac{1}{4n}\partial_\lambda W\,, \qquad \nu'_\rho = \frac{1}{4k}\partial_\nu W\,, \qquad \varphi'_\rho = -\frac{1}{4}\partial_\varphi W\,. \tag{2.14}$$

In order to see how the above method works, we briefly review some of the known results obtained by the BPS equations.

Let us consider the case where $N = Q = N_f = 0$ in which the potential (2.12) will have an over all factor of $e^{-2\varphi}$. Thus one may take the following ansatz for the superpotential [21]

$$W = 4e^{-\varphi} w(\lambda, \nu)\,. \tag{2.15}$$

We consider two distinct cases of $k = 0, 1$. For $k = 0$, the relation (2.13) is solved by $w(\lambda) = \pm\sqrt{c}$. With this choice, the superpotential will be flat in the λ direction and a free parameter will appear in the solution, λ_0. This parameter, however, has no physical significance because it can be absorbed by a rescaling of the $n = 5$ dimensional space. The BPS equation for ϕ is solved by $\phi = \pm\frac{\sqrt{c}}{2}\tau$. The resulting solution will be a six dimensional flat space with linear dilaton.

There is a second choice for W namely $w(\lambda) = \pm\sqrt{c}\cosh(\sqrt{5}\lambda)$. The BPS equations are solved by

$$e^\lambda = \left[\tanh\left(\frac{1}{2}\sqrt{c}\tau\right)\right]^{1/\sqrt{5}}, \qquad e^{2\phi} = \frac{1}{a}\frac{[\tanh(\frac{1}{2}\sqrt{c}\tau)]^{\sqrt{5}}}{\sinh(\sqrt{c}\tau)}\,, \tag{2.16}$$

where a is a constant. The solution has a naked singularity at $\tau = 0$ and for $\tau \to \infty$ it reduces to flat space with a linear dilaton.

For $k = 1$, $w = \sqrt{c}$ is still a solution, though in this case W is flat both in λ and ν directions and the latter parameterizes the solutions by the radius of S^1. The dilaton is found as $\phi = \frac{\sqrt{c}}{2}\tau + \phi_0$ and the resulting geometry will be a cylinder.

As a second solution for W one may take

$$w(\lambda, \nu) = \sqrt{c}\cosh\left(\frac{q\lambda + l\nu}{\sqrt{\frac{q^2}{4} + l^2}}\right), \tag{2.17}$$

where q and l are two real numbers. For $q = 0$ one finds

$$\lambda = \lambda_0\,, \qquad e^\nu = \tanh\left(\frac{1}{2}\sqrt{c}\tau\right), \qquad e^{2\phi} = \frac{1}{2a}\frac{1}{\cosh^2(\frac{1}{2}\sqrt{c}\tau)}\,, \tag{2.18}$$

and a is a constant. This solution is known as the "cigar" background. For general q and l one gets [22]

$$e^\lambda = \left[\coth\left(\frac{1}{2}\sqrt{c}\tau\right)\right]^{\frac{\frac{q}{4}}{\sqrt{\frac{q^2}{4}+l^2}}}, \qquad e^\nu = \left[\coth\left(\frac{1}{2}\sqrt{c}\tau\right)\right]^{\frac{l}{\sqrt{\frac{q^2}{4}+l^2}}}. \tag{2.19}$$

Requiring regularity at $\tau = 0$ one needs to have $\frac{q}{l} = -\frac{3}{4}$. The dilaton is identical to the one for the cigar solution.

Finally, it is interesting to note that in the absence of N_f, N and Q, the action (2.4) is invariant under the transformation $\nu \to -\nu$ and $\phi \to \phi - \nu$. This is nothing but T-duality by which one can generate new solutions. As an example, applying these transformations to the cigar solution one arrives at a new background known as the "trumpet" solution [22].

The inclusion of space filling branes can result in serious modifications in the problem. For example in [20] it was shown that in presence of these pairs a solution of the form $AdS_5 \times S^1$ is obtained in six dimensions whereas in their absence, as shown by [22], the $AdS_5 \times S^1$ and $AdS_3 \times S^3$ solutions are not possible.

In the following sections we will further study the effects of these space filling brane pairs with and without nonzero N or Q. Since we are looking for those backgrounds which correspond to field theories at their conformal fixed point, we will mainly be interested in solutions with a constant dilaton. Therefore the third equation in (2.6) requires that

$$2c - (n+1-k)N^2 e^{-2k\nu+2\phi} + 2(k-1)Q^2 e^{-2k\nu} - (n+k+3)N_f e^{\phi} = 0\,. \tag{2.20}$$

This relation will be our starting point for finding solutions in the following sections.

3. Solutions with N_f space filling uncharged branes

In this section we shall study our first example where we will turn on N_f space filling uncharged branes and set all other charges to zero. In this case by making use of the equation (2.20) one can find the dilaton which is

$$e^{\phi} = \frac{2c}{(n+k+3)N_f} = \frac{1}{N_f}\,. \tag{3.1}$$

Looking at the the potential (2.12) we realize that the two cases $k = 0, 1$ are special because the curvature contribution from S^k vanishes for them. We shall study these two cases separately. For other values of k we could not solve the equations.

$k = 0$. When $k = 0$, the only equation one should solve is the one for λ. It is useful to rewrite the second derivatives in the equations of motion with respect to τ instead of ρ.[5] In this notation, taking into account that $\varphi' = -5\lambda'$, one arrives at the following equation for λ

$$\lambda'' + 5\lambda'^2 = \frac{N_f}{2} e^{\phi}\,. \tag{3.2}$$

Here ϕ is given by (3.1). This equation can be solved for λ and the final solution is

$$\lambda = \frac{\tau}{\sqrt{10}} + \lambda_0\,. \tag{3.3}$$

Therefore the background geometry is given by

$$ds^2 = d\tau^2 + e^{2\tau/\sqrt{10}} dx_5^2\,, \tag{3.4}$$

which is an AdS_6 background with $R^2_{AdS} = 10$. Note that in the above metric we have rescaled the coordinates $\vec{x}$ by $e^{2\lambda_0}$.

[5] From now on all the derivatives are written with respect to τ unless stated.

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It is worth noting that six dimensional string theory supports another AdS_6 solution which has a nonzero RR five form, no space filling uncharged D5-branes and no transverse sphere and is obtained as [22]

$$e^{\phi} = \frac{2}{\sqrt{3}N}\,, \qquad R^2_{AdS} = \frac{15}{2}\,. \tag{3.5}$$

We will see in section 4 that a third AdS_6 solution with $k = 0$ can be obtained when both nonzero RR five form and uncharged $D5$ branes are present.

$k = 1$. In this case it is convenient to introduce two new variables x and y as the following

$$x = \lambda - \nu\,, \qquad y = 4\lambda + \nu\,. \tag{3.6}$$

In terms of these new variables the equations of motion are

$$\begin{aligned} y'' + y'^2 &= \frac{5}{2}\,, \\ x'' + x'y' &= 0\,. \end{aligned} \tag{3.7}$$

It is easy to solve the above equation for y leading to

$$y = \sqrt{\frac{5}{2}}\tau + y_0\,. \tag{3.8}$$

One can then proceed to find x from the second equation. The simplest solution can be found by setting $x' = 0$ which together with the solution for y will give the following solution for λ and ν

$$\lambda = \frac{\tau}{\sqrt{10}} + \lambda_0\,, \qquad \nu = \frac{\tau}{\sqrt{10}} + \nu_0\,. \tag{3.9}$$

The resulting geometry is thus again AdS_6 with $R^2_{AdS} = 10$ but this time one of the coordinates is compact, i.e.

$$ds^2 = d\tau^2 + e^{2\tau/\sqrt{10}}(dx_4^2 + d\theta^2)\,. \tag{3.10}$$

Here we have rescaled $\vec{x}$ by $e^{2\lambda_0}$ and set $\nu_0 = 0$.

It is also possible to find a more general solution for this case which is

$$\begin{aligned} \lambda &= -\frac{\tau}{\sqrt{10}} + \frac{2}{5}\log\left(e^{\frac{5\tau}{\sqrt{10}}} - c_0\right) + \lambda_0 \\ \nu &= -\frac{\tau}{\sqrt{10}} + \log\left(e^{\frac{5\tau}{\sqrt{10}}} + c_0\right) - \frac{3}{5}\log\left(e^{\frac{5\tau}{\sqrt{10}}} - c_0\right) + \nu_0\,. \end{aligned} \tag{3.11}$$

This solution has an additional free parameter in comparison with the previous one and reduces to that for $c_0 = 0$. Alternatively, (3.10) is the $\tau \to \infty$ limit of this solution. For $x' \neq 0$ we were not able to solve the equations.

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4. Solutions with nonzero N and N_f

In this section we shall study cases with a nonzero RR form in the presence of N_f space filling uncharged branes. The constant dilaton condition leads to the following equation

$$2c-(n+1-k)N^2e^{-2k\nu+2\phi}-(n+k+3)N_fe^{\phi}=0\,. \tag{4.1}$$

We recognize two different possibilities; if $n+1-k\neq 0$ then ν necessarily has to be a constant but for $n+1-k=0$ ($k=3$), ν may or may not be a constant. We shall consider these cases separately. From the above equation one can see that for $N_f=0$ a constant dilaton solution is not possible for $k=3$ but with a nonzero N_f this limitation is removed.

$k=0$. In this case the equation (4.1) can be easily solved for the dilaton

$$e^{\phi}=\frac{2}{N_f}\frac{1}{1+\sqrt{1+3N^2/N_f^2}}\,. \tag{4.2}$$

The other parameter can be obtained by solving the equation for λ. Doing so we find an AdS_6 solution with radius

$$R^2_{AdS}=5\frac{\left(1+\sqrt{1+3N^2/N_f^2}\right)^2}{1+2\frac{N^2}{N_f^2}+\sqrt{1+3N^2/N_f^2}}\,. \tag{4.3}$$

This is our first example in which the AdS radius has N and N_f dependence. Of course it depends only on the ratio of these numbers N/N_f. As a result this solution represents an AdS_6 solution with one parameter. It is also interesting to note that both $N_f\to 0$ and $N\to 0$ limits are smooth and the resulting solutions are those presented in (3.5) and (3.3) respectively.

$k=1$. This case has been studied in [20] and, for completeness, we present its final result. The solution turns out to $AdS_5\times S^1$ with the following radii for AdS_5 and S^1 spaces

$$R^2_{AdS}=6\,,\qquad R^2_{S^1}=\frac{2N^2}{3N_f^2}\,. \tag{4.4}$$

The Dilaton is also given by

$$e^{\phi}=\frac{2}{3N_f}\,. \tag{4.5}$$

It has been argued that in order to get a stable solution one needs to restrict N/N_f to values (~ 1) which correspond to the conformal window of the dual theory.

$k=2$. The constant dilaton condition (4.1) in this case, is solved by

$$e^{\phi}=\frac{2x-1}{x}\frac{2}{5N_f}\,, \tag{4.6}$$

where $x \equiv e^{2\nu} = R^2_{S^2}$ and is found from the equation for ν

$$x^4 + 2x^3 - \frac{1}{5}(\frac{N}{N_f})^2(2x-1)^2 = 0\,. \tag{4.7}$$

The equation for λ is easily solved and results in an $AdS_4 \times S^2$ background with

$$R^2_{AdS} = \frac{15x}{4x+3}\,. \tag{4.8}$$

It is worth noting that since x is a function of N/N_f the AdS radius is also a function of that and therefore a one parameter $AdS_4 \times S^2$ solution is obtained.

$k = 3$. We first find the constant dilaton from (4.1)

$$e^{\phi} = \frac{1}{N_f}\,. \tag{4.9}$$

The remaining equations of motion for ν and λ will read

$$\begin{aligned}
\lambda'' + 2\lambda'^2 + 3\lambda'\nu' &= \frac{1}{2}\frac{N^2}{N_f^2}e^{-6\nu} + \frac{1}{2}\,,\\
\nu'' + 3\nu'^2 + 2\lambda'\nu' &= 2e^{-2\nu} - \frac{1}{2}\frac{N^2}{N_f^2}e^{-6\nu} + \frac{1}{2}\,.
\end{aligned} \tag{4.10}$$

As we have mentioned in section 2, in this case one can choose ν to be either a constant or not. If we take it to be a constant, it is found from the second equation above

$$x^3 + 4x^2 - \frac{N^2}{N_f^2} = 0\,, \tag{4.11}$$

where now $x \equiv e^{2\nu} = R^2_{S^3}$. This relation shows that x is a function of N/N_f. It is now easy to solve the equation for λ and for the final solution we an $AdS_3 \times S^3$ with

$$R^2_{AdS} = \frac{2x}{x+2}\,. \tag{4.12}$$

Therefore, as in the previous cases, we find a one parameter solution. We note that similar to the $AdS_5 \times S^1$ case, this solution exists only if the space filling uncharged D-brane are present which means that the $N_f \to 0$ limit is not smooth. For non constant ν we were not able to solve the equations.

5. Solutions with nonzero N_f and NS two form

Let us now study the gravity solution when we have self-dual NS1 brane. This corresponds to the situation when we have a nonzero NSNS two for, $B_{\mu\nu}$. We consider the case with nonzero $B_{\mu\nu}$ in the present of N_f space filling uncharged brane and still look for a solution with constant dilaton. As stated before, such solutions are only relevant when $k = 3$. The constant dilaton condition reads

$$2c = 8N_f e^{\phi} - 4Q^2 e^{-6\nu}\,. \tag{5.1}$$

This relation implies that ν also has to be a constant. The equation for ν will read

$$x^3 + 4x^2 - \frac{3}{2}Q^2 = 0\,, \tag{5.2}$$

where again $x \equiv e^{2\nu} = R^2_{S^3}$ and is a function of Q. The dilaton is also found as

$$e^{\phi} = \frac{x+1}{x}\frac{4}{3N_f}\,. \tag{5.3}$$

Once we solve the equation for λ we find that the solution will be $AdS_3 \times S^3$ with

$$R^2_{AdS} = \frac{3x}{1+x}\,. \tag{5.4}$$

We note that here again we find a one parameter $AdS_3 \times S^3$ solution. This parameter is now given by Q. We note also that the $N_f \to 0$ limit is not smooth showing that this solution exists only in the presence of space filling uncharged branes.

6. Constant dilaton solutions from BPS equations

In the previous sections we obtained gravity solutions of six dimensional noncritical string in the form of $AdS_{n+1} \times S^{5-n}$ for $n = 2, 3, 4, 5$ using second order differential equations which are the equations of motion. As we mentioned before there is a method for finding solutions using first order differential equation known as BPS equations.

We note however that not all the solutions of the second order equations can necessarily be obtained form the first order equations. In fact satisfying the BPS equations is a necessary condition for the solution to be supersymmetric.

In this section we shall show that all our solutions, except one, can indeed be obtained from the BPS equations and therefore have a good chance to be supersymmetric. The exception is the (3.11) solution which is very similar to non extremal brane backgrounds with c_0 playing the role of the horizon radius. It is therefore natural for such a solution not to be supersymmetric and thus one cannot obtain it from BPS equations.

Our calculations are in parallel lines with those of [22]. Knowing that we are looking for a constant dilaton solution we can find the good variables for the superpotential in terms of which the calculations find a simple form. It is important to note that the solutions we are looking for do not require the superpotential to be independent of the dilaton and in fact this dependence is necessary because otherwise the constant dilaton would appear as a free parameter in the solution. Instead, we look for superpotentials that have an extremum at a certain value for the dilaton and thus the constant value for the dilaton is fixed.

Let us first study the solutions of section 3 for the case of $k = 1$. The good variables for this case are

$$x = \lambda - \nu\,, \qquad y = 4\lambda + \varphi + \nu\,, \qquad z = 4\lambda + 5\varphi + \nu\,. \tag{6.1}$$

Note that $y = 2\phi$ and the superpotential we are looking for must have an extremum at $y = y_0 = -2\ln N_f$. We also know from the solution that x has to be a constant as well. The field z has been chosen in a way that the metric G^{ab} defined in (2.9) is diagonal. In terms of our new variables the potential (2.12) takes the following form

$$V = e^{-z/2}\left(-ce^{\frac{y}{2}} + 2N_f e^{y}\right). \tag{6.2}$$

Therefore we will consider the following ansatz for the superpotential

$$W(x,y,z) = e^{-z/4} w(y). \tag{6.3}$$

Now putting everything together the relation (2.9) becomes

$$-\frac{5}{4} w(y)^2 + 4\partial_y w(y)^2 = 16\left(-ce^{\frac{y}{2}} + 2N_f e^{y}\right). \tag{6.4}$$

It is easy to check that V has an extremum at $y_0 = -2\ln N_f$ which, together with the above equation, tells us that $\partial_y w$ also has to vanish at y_0. Therefore the BPS equations will become

$$x' = 0\,, \qquad y'|_{y=y_0} = 0\,, \qquad z'|_{y=y_0} = -\frac{20}{\sqrt{10}}. \tag{6.5}$$

This means that

$$\lambda'|_{y=y_0} = \nu'|_{y=y_0} = \frac{1}{\sqrt{10}}, \tag{6.6}$$

which obviously produces the solution we were looking for.

Next we consider the remaining solutions which have an additional constant namely $\nu = \nu_0$. We therefore, following [22], parameterize the problem in terms of ν and the following two variables

$$x = \frac{1}{n-1}(\lambda + \varphi)\,, \qquad y = \frac{1}{n-1}(n\lambda + \varphi)\,, \tag{6.7}$$

where the overall numerical factors are chosen for convenience. The potential (2.12) is written as

$$V = e^{-2nx} V(y,\nu)\,, \tag{6.8}$$

where

$$V(y,\nu) = -ce^{2y} - k(k-1)e^{2y-2\nu} + N^2 e^{(n+1)y-k\nu} + Q^2 e^{2y-2k\nu} + 2N_f e^{\frac{3+n}{2}y + \frac{k}{2}\nu}. \tag{6.9}$$

We thus choose the following ansatz for the superpotential

$$W(x,y,\nu) = e^{-nx} w(y,\nu)\,, \tag{6.10}$$

and the relation (2.9) becomes

$$-\frac{n}{n-1} w^2 + \frac{1}{k}\partial_\nu w^2 + \frac{1}{n-1}\partial_y w^2 = 16 V(y,\nu). \tag{6.11}$$

Extremizing $V(y,\nu)$ will give us the constant values of y_0 and ν_0

$$\partial_y V(y,\nu)|_{y=y_0,\nu=\nu_0} = \partial_\nu V(y,\nu)|_{y=y_0,\nu=\nu_0} = 0. \tag{6.12}$$

The relation (6.11) tells us that $\partial_\nu w$ and $\partial_y w$ vanish at ν_0 and y_0 and that $w_0^2 \equiv w^2(y_0,\nu_0) = 16\frac{1-n}{n} V(y_0,\nu_0)$. Finally, the BPS equation for λ is

$$\lambda' = \left(\frac{V(y_0,\nu_o)}{n(1-n)}\right)^{1/2} e^{-y_0}. \tag{6.13}$$

It is now straight forward to check that all the solutions we have obtained so far can be obtained by the above formulation.

	$N_f \neq 0$	$N \neq 0$	$N, N_f \neq 0$
$k=0$	AdS_6	AdS_6	AdS_6
$k=1$	AdS_6	-	$AdS_5 \times S^1$
$k=2$		$AdS_4 \times S^2$	$AdS_4 \times S^2$
$k=3$		-	$AdS_3 \times S^3$

Table 1: AdS solutions in six dimensional noncritical string theory.

7. Conclusions

In this paper we have studied six dimensional noncritical string theory by looking at its low energy effective supergravity action. We looked for NS and D brane solutions of the equations of motion with one nonzero charge at a time. Our motivation was looking for backgrounds which can be considered as the gravity duals of conformal field theories. We therefore focused on constant dilaton solutions.

This problem has some features which are characteristics of noncritical theories. In the absence of branes, the equations never yield solutions with a constant dilaton. The famous examples are the linear dilaton and cigar solutions. Once branes are included, under certain circumstances, constant dilaton solutions are found. These solutions have the general form of $AdS_p \times S^q$. The situation is completely the opposite in critical theories where in the absence of branes the dilaton is a constant while the brane solutions generically yield a non constant dilaton. A famous counter example of course is the $D3$ brane of IIB.

Things are changed considerably when we add a new ingredient, uncharged space filling branes, to the noncritical theory. Firstly, one no longer needs nonzero NS or RR charges, or equivalently additional branes, to find a constant dilaton solution. We have given examples of this in section 3. Secondly, some of the brane solutions with constant dilaton which were impossible previously can now be obtained. Examples are $AdS_5 \times S^1$ of [20] and $AdS_3 \times S^3$ obtained in section 4 of the present work. Finally, the previously obtained $AdS_p \times S^q$ solutions are now just modified by a new constant value for the dilaton and new radii for the two AdS_p and S^q spaces. Examples of these cases are also given in section 4. We summarize all these solutions in table 1.

The $AdS_p \times S^q$ solutions we have found all share the general feature that the curvature of the AdS space and the radius of the transverse sphere are constant. This is expected and is another characteristic of noncritical theories. It is in fact the property that makes them useful for studying field theories with isolated conformal fixed points. The constant curvatures of our solutions are all of order one in the string scale which makes higher α' corrections necessary. As stated before, these corrections are not supposed to change the general form of the solutions.

One may also find AdS black hole solutions in this theory. For example the $AdS_5 \times S^1$ black hole solution is obtained as

$$ds^2 = \frac{R_{AdS}^2}{u^2}\frac{du^2}{1-\frac{u_0^4}{u^4}} - \frac{u^2}{R_{AdS}^2}\left(1-\frac{u_0^4}{u^4}\right)dt^2 + u^2 dx_i^2 + R_{S^1}^2 d\theta^2\,, \tag{7.1}$$

with the same radii as in the extremal case.

This solution can be used to study for example a confining gauge theory in three dimension. Since this solution has a free parameter namely N/N_f, one might suspect that it can be useful for having control over the glueball and KK masses of the confining theory. Four dimensional confining theory has recently been studied in [23] using noncritical AdS_6 black hole solution. It would be interesting to see how this black hole solution and the corresponding confining theory are modified once we use the AdS_6 solutions of sections 3 and 4 to construct the associated black hole solutions.

We have also given a second derivation of most of our results by solving the first order BPS equations. This is a necessary condition for the supersymmetry of solutions. The superpotential defined for this purpose has an extremum at the point of the solutions. Perturbations around these points can in principle determine the flows of the equations which asymptote in the one end to our solutions. To determine the other end one should presumably solve the equations numerically.

One would also like to know the form of the correspondence for the backgrounds under discussion. In order to do this one should determine the expansion parameters of the corresponding theories. Consider for example the $AdS_5 \times S^1$ solution. To validate the string tree level approximation N_f has to be very large ($e^{\phi} \sim 1/N_f$). This can be achieved in a more familiar form if we define the 't Hooft coupling of the field theory as in the conventional AdS/CFT correspondence by $\lambda = g_s N = N/N_f$. Therefore the weak string coupling limit translates into the usual 't Hooft or large N limit. By considering open string tachyons on the D5 branes, it was argued in [20] that $N/N_f \sim 1$. The sigma model expansion, on the other hand, is governed by a coupling which is of order of unity. As a result, the correspondence seems to be between classical noncritical string theory on this background and a large N field theory at its conformal fixed point where the 't Hooft coupling is of order of unity.

For the backgrounds we have found in section 4, the correspondence seems to hold in a similar way. Namely one would expect that $N/N_f \sim 1$ still holds, and therefore the 't Hooft couplings would still be of order of unity and also the string coupling should become very small in the large N limit.

One may ask what gravity could tell us about the degrees of freedom of the "dual gauge" theory. To estimate the degrees of freedom one may look at the gravity action which in all our cases turns out to be

$$\int \sqrt{g} e^{-2\phi} \mathcal{R} \sim N_f^2 f\left(\frac{N}{N_f}\right), \tag{7.2}$$

where the function $f(x)$ can be determined in each case. For example in the $AdS_5 \times S^1$ case it is given by $f(x) = x$. In general for $x \sim 1$ one obtains $f(x) \sim 1$. This suggests that the degrees of freedom of the dual gauge theory goes like N^2. Here we have used the fact that the 't Hooft coupling is of order of one.[6]

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